M-14-A

INDIAN MARITIME UNIVERSITY

(A CENTRAL UNIVERSITY, GOVERNMENT OF INDIA)

B.TECH (MARINE ENGINEERING)

DECEMBER2014/JANUARY 2015 END SEMSTER EXAMINATION

I SEMESTER

MATHEMATICS I (T 2102 / T 1102)

Time: 03.00 Hrs | Max. Marks: 100

Date:26-12-2014 | Pass Marks: 50

Part - A (3×10 = 30 **Marks**)

Compulsory Questions (The symbols have their usual meanings)

1. (a) If $y = \tan^{-1}\frac{1+x}{1-x}$, find y_n.

   (b) If u = log (tan x + tan y) , prove that

   Sin 2x $\frac{\partial u}{\partial x}$ + sin 2y $\frac{\partial u}{\partial y}$ = 2

   (c) Find the radius of curvature at any point of the curve

   $X = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$

   (d)) Find the maxima and minima of the function

   $X^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

   (e) If z = f(x,y) where $x = e^u \cos v$, $y = e^u \sin v$

   show that $(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2 = e^{-2u}\{(\frac{\partial f}{\partial u})^2 + (\frac{\partial f}{\partial v})^2\}$

   (f) Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$ and hence evaluate the same.

   (g) Show that the equations, 2x + 6y = -11 , 6x + 20y -6z = -3 , 6y – 18z = -1 are not consistent.

   (h) Given the vector field

   $\vec{v} = (x^2 - y^2 + 2xz)\,\hat{i} + (xz - xy + yz)\,\hat{j} + (z^2 + x^2)\,\hat{k}$

   Find curl $\vec{v}$. Show that the vectors given by curl $\vec{v}$ at P_0 (1, 2, -3) and P_1 (2,3,12) are orthogonal.

   (i) Evaluate $\int \frac{e^z}{z^2+\pi^2} dz$ where C is the circle $|z| = 4$.

   (j) Evaluate $\int_0^\infty \frac{x^a}{a^x} dx$,a > 1

Part - B (5×14 = 70 **Marks**)

Answer Any Five of the following

2. (a) If $\cos^{-1}(\frac{y}{b}) = \log(\frac{x}{a})^n$ then prove that $x^2 y_{n+2} + (2n+1)\,xy_{n+1} + 2n^2 y_n = 0$

   (b) Find the asymptotes to the curve

   $4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0$. 7+7

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3. (a) If $u = \sin^{-1}\sqrt{\frac{x^{\frac{1}{3}}+y^{\frac{1}{3}}}{x^{\frac{1}{12}}+y^{\frac{1}{12}}}}$ then prove that

$$X^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial x\partial y} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$$

(b) In a plane triangle ABC find the maximum value of cosA cosB cosC. 7+7

4.(a) Use the rule of differentiation under the sign of integration to evaluate $\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)}\,dx$

(b) The intercept of $\frac{x}{a}+\frac{y}{b}=1$ between the axis of co-ordinates revolves about the y – axis. Find the volume and surface area of the solid thus generated. 6+8

5.(a) Using the transformation $x+y = u$, $y = uv$ show that

$$\iint_c e^{\frac{y-x}{y+x}}\,dx\,dy = \frac{1}{4}\left(e - \frac{1}{e}\right)$$

Integration being taken over the area of the triangle bounded by the lines $x = 0$, $y = 0$, $x + y = 1$

(b) Evaluate $\iiint_V (x^2+y^2+z^2)\,dxdydz$ where $v = \{0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1\}$. 7+7

6. (a) Show that curl grad $\vec{f} = \vec{0}$ where $f = x^2 y + 2xy + z^2$.

(b) A fluid motion is given by

$\vec{v} = (y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\hat{j} + (xy\cos z + y^2)\hat{k}$.

Is the motion irrotational? If so, find the velocity potential. 7 + 7

7. Determine the eigen values and eigen vectors of the matrix 14

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

8. (a) Expand $f(z) = \frac{z+1}{(z-3)(z-4)}$ in a Taylor's series about the point $z = 2$.

(b) Find the value of $\int_c \frac{z^2+2z+2}{(z-1)^2(z-2)}\,dz$ where $C : |Z| = 6$. 7+7

9. Find the curve connecting two points (not on a vertical line) such that a particle sliding down this curve under gravity (in absence of resistance) from one point to another reaches in the shortest time. 14

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